# Package 'PLMIX'

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Type Package

Title Bayesian Analysis of Finite Mixture of Plackett-Luce Models

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Description Fit finite mixtures of Plackett-Luce models for partial top rank-

ings/orderings within the Bayesian framework. It provides MAP point estimates via EM algorithm and posterior MCMC simulations via Gibbs Sampling. It also fits MLE as a special case of the noninformative Bayesian analysis with vague priors. In addition to inferential techniques, the package assists other fundamental phases of a model-based analysis for partial rankings/orderings, by including functions for data manipulation, simulation, descriptive summary, model selection and goodness-of-fit evaluation. Main references on the methods are Mollica and Tardella (2017) <doi:10.1007/s11336-016-9530-0> and Mollica and Tardella (2014) <doi:10.1002/sim.6224>.

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# Encoding UTF-8

**Imports** Rcpp (>= 1.0.5), abind (>= 1.4-5), foreach (>= 1.4.4), ggplot2 (>= 2.2.1), ggmcmc (>= 1.2), coda (>= 0.19-1), reshape2 (>= 1.4.3), rcdd (>= 1.2), gridExtra (>= 2.3), MCMCpack (>= 1.4-2), label.switching (>= 1.6), PlackettLuce (>= 0.2-3), radarchart (>= 0.3.1), methods, stats, utils

#### LinkingTo Rcpp

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as.top\_ordering

### Description

Attempt to coerce the input data into a top-ordering dataset.

# Usage

```
as.top_ordering(
   data,
   format_input = NULL,
   aggr = NULL,
   freq_col = NULL,
   ties_method = "random",
   ...
)
```

### Arguments

data	An object containing the partial sequences to be coerced into an object of class top_ordering. The following classes are admissible for data: numeric matrix, data.frame, RandData from the rankdist package and rankings from the PlackettLuce package.
format_input	Character string indicating the format of the data input, namely "ordering" or "ranking". Used only when the class of the data argument is matrix or data frame. Default is NULL.
aggr	Logical: whether the data argument collects the distinct observed sequences with the corresponding frequencies (aggregated format). Used only when the class of the data aargument is matrix or data frame. Default is NULL.
freq_col	Integer indicating the column of the data argument containing the frequencies of the distinct observed sequences. Used only when the class of the data argument is matrix or data frame and aggr argument is TRUE. Default is NULL.
ties_method	Character string indicating the treatment of sequences with ties (not used for data of class RankData). If "remove", the sequences with ties are removed before acting the coercion; if "random" (default), tied positions are re-assigned at random before acting the coercion.
	Further arguments passed to or from other methods (not used).

# Details

The coercion function as.top\_ordering tries to coerce the input data into an object of class top\_ordering after checking for possible partial sequences that do not satisfy the top-ordering requirements. If none of the supplied sequences satisfies the top-ordering conditions, an error message is returned. NA's in the input data are tacitly converted into zero entries.

An object of S3 class c("top\_ordering", "matrix").

#### Author(s)

Cristina Mollica and Luca Tardella

## References

Turner, H., Kormidis, I. and Firth, D. (2018). PlackettLuce: Plackett-Luce Models for Rankings. R package version 0.2-3. https://CRAN.R-project.org/package=PlackettLuce

Qian, Z. (2018). rankdist: Distance Based Ranking Models. R package version 1.1.3. https://CRAN.R-project.org/package=rankdist

### See Also

is.top\_ordering

```
## Coerce an object of class 'rankings' into an object of class 'top_ordering'
library(PlackettLuce)
RR <- matrix(c(1, 2, 0, 0,
4, 1, 2, 3,
2, 1, 1, 1,
1, 2, 3, 0,
2, 1, 1, 0,
1, 0, 3, 2), nrow = 6, byrow = TRUE)
RR_rank=as.rankings(RR)
RR_rank
as.top_ordering(RR_rank, ties_method="random")
## Coerce an object of class 'RankData' into an object of class 'top_ordering'
library(rankdist)
data(apa_partial_obj)
d_apa_top_ord=as.top_ordering(data=apa_partial_obj)
identical(d_apa,d_apa_top_ord)
## Coerce a data frame from the package prefmod into an object of class 'top_ordering'
library(prefmod)
data(carconf)
carconf_rank=carconf[,1:6]
carconf_top_ord=as.top_ordering(data=carconf_rank,format_input="ranking",aggr=FALSE)
identical(d_carconf,carconf_top_ord)
## Coerce a data frame from the package pmr into an object of class 'top_ordering'
library(pmr)
data(big4)
head(big4)
big4_top_ord=as.top_ordering(data=big4,format_input="ranking",aggr=TRUE,freq_col=5)
head(big4_top_ord)
```

bicPLMIX

### Description

Compute BIC value for the MLE of a mixture of Plackett-Luce models fitted to partial orderings.

### Usage

```
bicPLMIX(max_log_lik, pi_inv, G, ref_known = TRUE, ref_vary = FALSE)
```

# Arguments

max_log_lik	Maximized log-likelihood value.
pi_inv	An object of class top_ordering, collecting the numeric $N \times K$ data matrix of partial orderings, or an object that can be coerced with as.top_ordering.
G	Number of mixture components.
ref_known	Logical: whether the component-specific reference orders are known (not to be estimated). Default is TRUE.
ref_vary	Logical: whether the reference orders vary across mixture components. Default is FALSE.

# Details

The max\_log\_lik and the BIC values can be straightforwardly obtained from the output of the mapPLMIX and mapPLMIX\_multistart functions when the default noninformative priors are adopted in the MAP procedure. So, the bicPLMIX function is especially useful to compute the BIC value from the output of alternative MLE methods for mixtures of Plackett-Luce models implemented, for example, with other softwares.

The ref\_known and ref\_vary arguments accommodate for the more general mixture of Extended Plackett-Luce models (EPL), involving the additional reference order parameters (Mollica and Tardella 2014). Since the Plackett-Luce model is a special instance of the EPL with the reference order equal to the identity permutation  $(1, \ldots, K)$ , the default values of ref\_known and ref\_vary are set equal, respectively, to TRUE and FALSE.

# Value

A list of two named objects:

max_log_lik	The max_log_lik argument.
bic	BIC value.

### Author(s)

Cristina Mollica and Luca Tardella

### References

Mollica, C. and Tardella, L. (2017). Bayesian Plackett-Luce mixture models for partially ranked data. *Psychometrika*, **82**(2), pages 442–458, ISSN: 0033-3123, <doi:10.1007/s11336-016-9530-0>.

Mollica, C. and Tardella, L. (2014). Epitope profiling via mixture modeling for ranked data. *Statistics in Medicine*, **33**(21), pages 3738–3758, ISSN: 0277-6715, <doi:10.1002/sim.6224>.

Schwarz, G. (1978). Estimating the dimension of a model. *Ann. Statist.*, **6**(2), pages 461–464, ISSN: 0090-5364, <doi:10.1002/sim.6224>.

### See Also

mapPLMIX and mapPLMIX\_multistart

#### Examples

```
data(d_carconf)
K <- ncol(d_carconf)
MAP_mult <- mapPLMIX_multistart(pi_inv=d_carconf, K=K, G=3, n_start=2, n_iter=400*3)
bicPLMIX(max_log_lik=MAP_mult$mod$max_objective, pi_inv=d_carconf, G=3)$bic</pre>
```

## Equivalently
MAP\_mult\$mod\$bic

binary\_group\_ind Binary group membership matrix

### Description

Construct the binary group membership matrix from the multinomial classification vector.

#### Usage

```
binary_group_ind(class, G)
```

#### Arguments

class	Numeric vector of class memberships.
G	Number of possible different classes.

### Value

Numeric length(class)  $\times G$  matrix of binary group memberships.

### Author(s)

Cristina Mollica and Luca Tardella

### d\_apa

# Examples

binary\_group\_ind(class=c(3,1,5), G=6)

d\_apa

American Psychological Association Data (partial orderings)

### Description

The popular American Psychological Association dataset (d\_apa) contains the results of the voting ballots of the 1980 presidential election. A total of N = 15449 voters ranked a maximum of K = 5 candidates, conventionally classified as research psychologists (candidate 1 and 3), clinical psychologists (candidate 4 and 5) and community psychologists (candidate 2). The winner of the election was candidate 3. The dataset is composed of partial top orderings of varying lengths. Missing positions are denoted with zero entries.

### Format

Object of S3 class c("top\_ordering", "matrix") gathering a matrix of partial orderings with N = 15449 rows and K = 5 columns Each row lists the candidates from the most-liked (Rank\_1) to the least-liked (Rank\_5) in a given voting ballot.

### References

Mollica, C. and Tardella, L. (2017). Bayesian Plackett-Luce mixture models for partially ranked data. *Psychometrika*, **82**(2), pages 442–258, ISSN: 0033-3123, <doi: 10.1007/s11336-016-9530-0>.

Diaconis, P. W. (1988). Group representations in probability and statistics. *Lecture Notes-Monograph Series*, pages 94–96.

Diaconis, P. W. (1987). Spectral analysis for ranked data. Technical Report 282, Dept of Statistics, Stanford University.

```
data(d_apa)
head(d_apa)
```

```
## Subset of complete sequences
d_apa_compl=d_apa[rowSums(d_apa!=0)>=(ncol(d_apa)-1),]
head(d_apa_compl)
```

d\_carconf

#### Description

The Car Configurator dataset (d\_carconf) came up from a marketing study aimed at investigating customer preferences toward different car features. A sample of N = 435 customers were asked to construct their car by using an online configurator system and choose among K = 6 car modules in order of preference. The car features are labeled as: 1 = price, 2 = exterior design, 3 = brand, 4 = technical equipment, 5 = producing country and 6 = interior design. The survey did not require a complete ranking elicitation, therefore the dataset is composed of partial top orderings of varying lengths. Missing positions are denoted with zero entries.

### Format

Object of S3 class c("top\_ordering", "matrix") gathering a matrix of partial orderings with N = 435 rows and K = 6 columns. Each row lists the car features from the most important (Rank\_1) to the least important (Rank\_6) for a given customer.

### References

Mollica, C. and Tardella, L. (2017). Bayesian Plackett-Luce mixture models for partially ranked data. *Psychometrika*, **82**(2), pages 442–458, ISSN: 0033-3123, <doi: 10.1007/s11336-016-9530-0>.

Hatzinger, R. and Dittrich, R. (2012). Prefmod: An R package for modeling preferences based on paired comparisons, rankings, or ratings. *Journal of Statistical Software*, **48**(10), pages 1–31.

Dabic, M. and Hatzinger, R. (2009). Zielgruppenadaequate Ablaeufe in Konfigurationssystemen eine empirische Studie im Automobilmarkt - Partial Rankings. In Hatzinger, R., Dittrich, R. and Salzberger, T. (eds), *Praeferenzanalyse mit R: Anwendungen aus Marketing, Behavioural Finance und Human Resource Management*. Wien: Facultas.

```
data(d_carconf)
head(d_carconf)
```

```
## Subset of complete sequences
d_carconf_compl=d_carconf[rowSums(d_carconf!=0)>=(ncol(d_carconf)-1),]
head(d_carconf_compl)
```

d\_dublinwest

### Description

The Dublin West dataset (d\_dublinwest) contains the results of the voting ballots of the 2002 Irish general election from the Dublin West constituency. The Irish voting system allows voters to rank the candidates in order of preferences, rather than only specify the favorite one. In the Dublin West constituency, N = 29988 voters ranked a maximum of K = 9 candidates, labeled as: 1 = Bonnie R., 2 = Burton J., 3 = Doherty-Ryan D., 4 = Higgins J., 5 = Lenihan B., 6 = McDonald M., 7 = Morrissey T., 8 = Smyth J. and 9 = Terry S.. The dataset is composed of partial top orderings of varying lengths. Missing positions are denoted with zero entries.

#### Format

Object of S3 class c("top\_ordering", "matrix") gathering a partial orderings with N = 29988 rows and K = 9 columns. Each row lists the candidates from the most-liked (Rank\_1) to the least-liked (Rank\_9) in a given voting ballot.

### Source

The 2002 Dublin West data have been downloaded from https://preflib.github.io/PrefLib-Jekyll/ PrefLib: A Library for Preferences. In that repository, preferences with ties are also included. The original source was publicly available from the Dublin County Returning Officer at the following URL: https://dublincountyreturningofficer.com/.

### References

Mattei, N. and Walsh, T. (2013) PrefLib: A Library of Preference Data. *Proceedings of Third International Conference on Algorithmic Decision Theory* (ADT 2013). Springer, Lecture Notes in Artificial Intelligence, November 13-15, 2013.

Gormley, I. C. and Murphy, T. B. (2009). A grade of membership model for rank data. *Bayesian Analysis*, **4**(2), pages 65–295.

Gormley, I. C. and Murphy, T. B. (2008). Exploring Voting Blocs Within the Irish Electorate: A Mixture Modeling Approach. *Journal of the America Statistical Association*, **103**(483), pages 1014–1027.

```
data(d_dublinwest)
head(d_dublinwest)
```

```
## Subset of complete sequences
d_dublinwest_compl=d_dublinwest[rowSums(d_dublinwest!=0)>=(ncol(d_dublinwest)-1),]
head(d_dublinwest_compl)
```

d\_gaming

### Description

The Gaming Platforms dataset (d\_gaming) collects the results of a survey conducted on a sample of N = 91 Dutch students, who were asked to rank K = 6 gaming platforms in order of preference, namely: 1 = X-Box, 2 = PlayStation, 3 = PSPortable, 4 = GameCube, 5 = GameBoy and 6 = Personal Computer. The dataset is composed of complete orderings.

### Format

Object of S3 class c("top\_ordering", "matrix") gathering a matrix of complete orderings with N = 91 rows and K = 6 columns. Each row lists the gaming platforms from the most-liked (Rank\_1) to the least-liked (Rank\_6) for a given student.

### Source

The Gaming Platforms dataset in .csv format can be downloaded from http://qed.econ.queensu. ca/jae/2012-v27.5/fok-paap-van\_dijk/. The .csv files contains the preference data in ranking format and some covariates collected for each student.

## References

Fok, D., Paap, R. and Van Dijk, B. (2012). A Rank-Ordered Logit Model With Unobserved Heterogeneity In Ranking Capatibilities. *Journal of Applied Econometrics*, **27**(5), pages 831–846.

### Examples

data(d\_gaming)
head(d\_gaming)

d\_german

German Sample Data (complete orderings)

#### Description

The German Sample dataset (d\_german) is part of a comparative cross-sectional study on political actions and mass participation involving five Western countries. The dataset regards a sample of N = 2262 German respondents who were asked to rank K = 4 political goals in order of desirability, namely: 1 = maintaining order in the nation, 2 = giving people more say in the decisions of government, 3 = fighting rising prices and 4 = protecting freedom of speech. The dataset is composed of complete orderings.

#### d\_nascar

### Format

Object of S3 class c("top\_ordering", "matrix") gathering a matrix of complete orderings with N = 2262 rows and K = 4 columns. Each row lists the political goals from the most desiderable (Rank\_1) to the least desiderable (Rank\_4) for a given respondent.

#### References

Croon, M. A. (1989). Latent class models for the analysis of rankings. In De Soete, G., Feger, H. and Klauer, K. C. (eds), *New Developments in Psychological Choice Modeling*, pages 99–121. North-Holland: Amsterdam.

Barnes, S. H. et al. (1979). Political action. Mass participation in five Western democracies. London: Sage.

#### Examples

data(d\_german)
head(d\_german)

d\_nascar

NASCAR Data (partial orderings)

#### Description

The NASCAR dataset (d\_nascar) collects the results of the 2002 season of stock car racing held in the United States. The 2002 championship consisted of N = 36 races, with 43 car drivers competing in each race. A total of K = 87 drivers participated in the 2002 season, taking part to a variable number of races: some of them competed in all the races, some others in only one. The results of the entire 2002 season were collected in the form of top-43 orderings, where the position of the not-competing drivers in each race is assumed lower than the 43th, but undetermined. Missing positions are denoted with zero entries.

#### Format

Object of S3 class c("top\_ordering", "matrix") gathering a matrix of partial orderings with N = 36 rows and K = 87 columns. Each row lists the car drivers from the top position (Rank\_1) to the bottom one (Rank\_87) in a given race. Columns from the 44th to the 87th are filled with zeros, because only 43 drivers competed in each race.

#### References

Caron, F. and Doucet, A. (2012). Efficient Bayesian inference for Generalized Bradley-Terry models. *J. Comput. Graph. Statist.*, **21**(1), pages 174–196.

Guiver, J. and Snelson, E. (2009). Bayesian inference for Plackett-Luce ranking models. In Bottou, L. and Littman, M., editors, *Proceedings of the 26th International Conference on Machine Learning* - *ICML 2009*, pages 377–384. Omnipress.

Hunter, D. R. (2004). MM algorithms for Generalized Bradley-Terry models. *Ann. Statist.*, **32**(1), pages 384–406.

### Examples

```
data(d_nascar)
head(d_nascar)
## Compute the number of races for each of the 87 drivers
table(c(d_nascar[,1:43]))
## Identify drivers arrived last (43th position) in all the races
which(colSums(rank_summaries(d_nascar, format="ordering")$marginals[1:42,])==0)
## Obscure drivers 84, 85, 86 and 87 to get the reduced dataset
## with 83 racers employed by Hunter, D. R. (2004)
d_nascar_hunter=d_nascar[,1:83]
d_nascar_hunter[is.element(d_nascar_hunter,84:87)]=0
```

```
d_occup
```

Occupation Data (complete orderings)

#### Description

The Occupation dataset (d\_occup) came up from a survey conducted on graduates from the Technion-Insrael Institute of Tecnology. A sample of N = 143 graduates were asked to rank K = 10 professions according to the perceived prestige. The occupations are labeled as: 1 = faculty member, 2 = owner of a business, 3 = applied scientist, 4 = operations researcher, 5 = industrial engineer, 6 = manager, 7 = mechanical engineer, 8 = supervisor, 9 = technician and 10 = foreman. The dataset is composed of complete orderings.

# Format

Object of S3 class c("top\_ordering", "matrix") gathering a matrix of complete orderings with N = 143 rows and K = 10 columns. Each row lists the professions from the most-liked (Rank\_1) to the least-liked (Rank\_10) for a given graduate.

### References

Cohen, A. and Mallows, C. L. (1983). Assessing goodness of fit of ranking models to data. *Journal of the Royal Statistical Society: Series D (The Statistician)*, **32**(4), pages 361–374, ISSN: 0039-0526.

Cohen, A. (1982). Analysis of large sets of ranking data. *Communications in Statistics – Theory and Methods*, **11**(3), pages 235–256.

Goldberg, A. I. (1976). The relevance of cosmopolitan/local orientations to professional values and behavior. *Sociology of Work and Occupations*, **3**(3), pages 331–356.

### Examples

data(d\_occup)
head(d\_occup)

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d\_rice

### Description

The Rice Voting dataset (d\_rice) collects the results of the 1992 election of a faculty member to serve on the Presidential Search Committee in the Rice University. A total of N = 300 people casted their vote in the ballots by ranking the K = 5 candidates in the short list in a preferential manner. The dataset is composed of partial top orderings of varying lengths. Missing positions are denoted with zero entries.

# Format

Object of S3 class c("top\_ordering", "matrix") gathering a matrix of partial orderings with N = 300 rows and K = 5 columns. Each row lists the faculty members from the most-liked (Rank\_1) to the least-liked (Rank\_5) in a given voting ballot.

### References

Marcus, P., Heiser, W. J. and D'Ambrosio, A. (2013). Comparison of heterogeneous probability models for ranking data, Master Thesis, Leiden University.

Baggerly, K. A. (1995). Visual estimation of structure in ranked data, PhD thesis, Rice University.

### Examples

```
data(d_rice)
head(d_rice)
## Subset of complete sequences
d_rice_compl=d_rice[rowSums(d_rice!=0)>=(ncol(d_rice)-1),]
head(d_rice_compl)
```

fill_single_entries	Utility to fill in single missing entries of top-(K-1) sequences in partial
	ordering/ranking datasets

### Description

Utility to fill in single missing entries of top-(K-1) sequences in partial ordering/ranking datasets

### Usage

fill\_single\_entries(data)

#### Arguments

data

Numeric data matrix of partial sequences.

# Value

Numeric data matrix of partial sequences in the same format of the input data with possible single missing entries filled.

# Author(s)

Cristina Mollica and Luca Tardella

freq\_to\_unit Individual rankings/orderings from the frequency distribution

### Description

Construct the dataset of individual rankings/orderings from the frequency distribution of the distinct observed sequences.

### Usage

```
freq_to_unit(freq_distr)
```

### Arguments

freq\_distr Numeric matrix of the distinct observed sequences with the corresponding frequencies indicated in the last (K + 1)-th column.

# Value

Numeric  $N \times K$  data matrix of observed individual sequences.

### Author(s)

Cristina Mollica and Luca Tardella

```
library(e1071)
K <- 4
perm_matrix <- permutations(n=K)
freq_data <- cbind(perm_matrix, sample(1:factorial(K)))
freq_data
freq_to_unit(freq_distr=freq_data)</pre>
```

gibbsPLMIX

# Description

Perform Gibbs sampling simulation for a Bayesian mixture of Plackett-Luce models fitted to partial orderings.

# Usage

```
gibbsPLMIX(
    pi_inv,
    K,
    G,
    init = list(z = NULL, p = NULL),
    n_iter = 1000,
    n_burn = 500,
    hyper = list(shape0 = matrix(1, nrow = G, ncol = K), rate0 = rep(0.001, G), alpha0 =
        rep(1, G)),
    centered_start = FALSE
)
```

# Arguments

pi_inv	An object of class top_ordering, collecting the numeric $N \times K$ data matrix of partial orderings, or an object that can be coerced with as.top_ordering.
К	Number of possible items.
G	Number of mixture components.
init	List of named objects with initialization values: z is a numeric $N \times G$ matrix of binary mixture component memberships; p is a numeric $G \times K$ matrix of component-specific support parameters. If starting values are not supplied (NULL), they are randomly generated with a uniform distribution. Default is NULL.
n_iter	Total number of MCMC iterations.
n_burn	Number of initial burn-in drawings removed from the returned MCMC sample.
hyper	List of named objects with hyperparameter values for the conjugate prior spec- ification: shape0 is a numeric $G \times K$ matrix of shape hyperparameters; rate0 is a numeric vector of G rate hyperparameters; alpha0 is a numeric vector of G Dirichlet hyperparameters. Default is vague prior setting.
centered_start	Logical: whether a random start whose support parameters and weights should be centered around the observed relative frequency that each item has been ranked top. Default is FALSE. Ignored when init is not NULL.

### Details

The size *L* of the final MCMC sample is equal to n\_iter-n\_burn.

# Value

A list of S3 class gsPLMIX with named elements:

W	Numeric $L \times G$ matrix with MCMC samples of the mixture weights.
Р	Numeric $L \times (G * K)$ matrix with MCMC samples of the component-specific support parameters.
log_lik	Numeric vector of $L$ posterior log-likelihood values.
deviance	Numeric vector of $L$ posterior deviance values ( $-2*log_lik$ ).
objective	Numeric vector of $L$ objective function values (that is the kernel of the log-posterior distribution).
call	The matched call.

#### Author(s)

Cristina Mollica and Luca Tardella

### References

Mollica, C. and Tardella, L. (2017). Bayesian Plackett-Luce mixture models for partially ranked data. *Psychometrika*, **82**(2), pages 442–458, ISSN: 0033-3123, <doi:10.1007/s11336-016-9530-0>.

# Examples

```
data(d_carconf)
GIBBS <- gibbsPLMIX(pi_inv=d_carconf, K=ncol(d_carconf), G=3, n_iter=30, n_burn=10)
str(GIBBS)
GIBBS$P
GIBBS$W</pre>
```

gsPLMIX\_to\_mcmc MCMC class objects from the Gibbs sampling simulations of a Bayesian mixture of Plackett-Luce models

# Description

Coerce the Gibbs sampling simulations for a Bayesian mixture of Plackett-Luce models into an mcmc class object.

#### Usage

```
gsPLMIX_to_mcmc(gsPLMIX_out)
```

# Arguments

gsPLMIX\_out Object of class gsPLMIX returned by the gibbsPLMIX function.

# is.top\_ordering

# Details

gsPLMIX\_to\_mcmc attemps to coerce its argument by recalling the as.mcmc function of the coda package.

# Value

An mcmc class object.

### Author(s)

Cristina Mollica and Luca Tardella

### References

Plummer, M., Best, N., Cowles, K. and Vines, K. (2006). CODA: Convergence Diagnosis and Output Analysis for MCMC, *R News*, **6**, pages 7–11, ISSN: 1609-3631.

### See Also

as.mcmc

# Examples

```
data(d_carconf)
GIBBS <- gibbsPLMIX(pi_inv=d_carconf, K=ncol(d_carconf), G=3, n_iter=30, n_burn=10)</pre>
```

## Coerce the posterior samples into an mcmc class object gsPLMIX\_to\_mcmc(GIBBS)

is.top\_ordering Top-ordering datasets

#### Description

Check the consistency of partial ordering data with a top-ordering dataset.

### Usage

```
is.top_ordering(data, ...)
```

#### Arguments

data	An object containing the partial orderings whose consistency with a top-ordering
	dataset has to be tested. The following classes are admissible for data: numeric
	matrix, data.frame, RandData from the rankdist package and rankings
	from the PlackettLuce package.
	Further arguments passed to or from other methods (not used).

### Details

The argument data requires the partial sequences expressed in ordering format. When the value of is.top-ordering is FALSE, the membership function returns also a message with the conditions that are not met for the data to be a top-ordering dataset. NA's in the input data are tacitly converted into zero entries.

### Value

Logical: TRUE if the data argument is consistent with a top-ordering dataset (with a possible warning message if the supplied data need a further treatment with the coercion function as.top\_ordering before being processed with the core functions of **PLMIX**) and FALSE otherwise.

### Author(s)

Cristina Mollica and Luca Tardella

### References

Turner, H., Kormidis, I. and Firth, D. (2018). PlackettLuce: Plackett-Luce Models for Rankings. R package version 0.2-3. https://CRAN.R-project.org/package=PlackettLuce

Qian, Z. (2018). rankdist: Distance Based Ranking Models. R package version 1.1.3. https://CRAN.R-project.org/package=rankdist

#### Examples

## A toy example of data matrix not satisfying the conditions to be a top-ordering dataset toy\_data=rbind(1:5,

c(0,4,3,2,1), c(4,3.4,2,1,5), c(2,3,0,0,NA), c(4,4,3,2,5), c(3,5,4,2,6), c(2,-3,1,4,5), c(2,0,1,4,5), c(2,3,1,1,1), c(2,3,0,4,0))

is.top\_ordering(data=toy\_data)

## A dataset from the StatRank package satisfying the conditions to be a top-ordering dataset library(StatRank) data(Data.Election9) is.top\_ordering(data=Data.Election9) label\_switchPLMIX

Label switching adjustment of the Gibbs sampling simulations for Bayesian mixtures of Plackett-Luce models

# Description

Remove the label switching phenomenon from the MCMC samples of Bayesian mixtures of Plackett-Luce models with G > 1 components.

### Usage

```
label_switchPLMIX(
   pi_inv,
   seq_G,
   MCMCsampleP,
   MCMCsampleW,
   MAPestP,
   MAPestW,
   parallel = FALSE
)
```

### Arguments

pi_inv	An object of class top_ordering, collecting the numeric $N \times K$ data matrix of partial orderings, or an object that can be coerced with as.top_ordering.
seq_G	Numeric vector with the number of components of the Plackett-Luce mixtures to be assessed.
MCMCsampleP	List of size length(seq_G), whose generic element is a numeric $L \times (G * K)$ matrix with the MCMC samples of the component-specific support parameters to be processed.
MCMCsampleW	List of size length(seq_G), whose generic element is a numeric $L \times G$ matrix with the MCMC samples of the mixture weights to be processed.
MAPestP	List of size length(seq_G), whose generic element is a numeric $G \times K$ matrix with the MAP estimates of the component-specific support parameters to be used as a pivot in the PRA method (see 'Details').
MAPestW	List of size length(seq_G), whose generic element is a numeric vector with the MAP estimates of the $G$ mixture weights to be used as a pivot in the PRA method (see 'Details').
parallel	Logical: whether parallelization should be used. Default is FALSE.

### Details

The label\_switchPLMIX function performs the label switching adjustment of the MCMC samples via the Pivotal Reordering Algorithm (PRA) described in Marin et al (2005), by recalling the pra function from the label.switching package.

### Value

A list of named objects:

final_sampleP	List of size length(seq_G), whose generic element is a numeric $G \times K \times L$ array with the MCMC samples of the component-specific support parameters adjusted for label switching.
final_sampleW	List of size length(seq_G), whose generic element is a numeric $L \times G$ matrix with the MCMC samples of the mixture weights adjusted for label switching.

# Author(s)

Cristina Mollica and Luca Tardella

### References

Mollica, C. and Tardella, L. (2017). Bayesian Plackett-Luce mixture models for partially ranked data. *Psychometrika*, **82**(2), pages 442–458, ISSN: 0033-3123, <doi:10.1007/s11336-016-9530-0>.

Papastamoulis, P. (2016). label.switching: An R Package for Dealing with the Label Switching Problem in MCMC Outputs. *Journal of Statistical Software*, **69**(1), pages 1–24, <doi:10.18637/jss.v069.c01>.

Marin, J. M., Mengersen, K. and Robert, C.P. (2005). Bayesian modelling and inference on mixtures of distributions. *Handbook of Statistics* (25), D. Dey and C.R. Rao (eds). Elsevier-Sciences.

#### See Also

pra

```
data(d_carconf)
K <- ncol(d_carconf)</pre>
## Fit 1- and 2-component PL mixtures via MAP estimation
MAP_1 <- mapPLMIX_multistart(pi_inv=d_carconf, K=K, G=1,</pre>
                              n_start=2, n_iter=400*1)
MAP_2 <- mapPLMIX_multistart(pi_inv=d_carconf, K=K, G=2,</pre>
                               n_start=2, n_iter=400*2)
MAP_3 <- mapPLMIX_multistart(pi_inv=d_carconf, K=K, G=3,</pre>
                               n_start=2, n_iter=400*3)
mcmc_iter <- 30</pre>
burnin <- 10
## Fit 1- and 2-component PL mixtures via Gibbs sampling procedure
GIBBS_1 <- gibbsPLMIX(pi_inv=d_carconf, K=K, G=1, n_iter=mcmc_iter,</pre>
                       n_burn=burnin, init=list(p=MAP_1$mod$P_map,
                       z=binary_group_ind(MAP_1$mod$class_map,G=1)))
GIBBS_2 <- gibbsPLMIX(pi_inv=d_carconf, K=K, G=2, n_iter=mcmc_iter,</pre>
                       n_burn=burnin, init=list(p=MAP_2$mod$P_map,
```

label\_switchPLMIX\_single Label switching adjustment for mixtures of Plackett-Luce models

### Description

Remove the label switching phenomenon from the MCMC samples of Bayesian mixtures of Plackett-Luce models with a different number of components.

### Usage

label\_switchPLMIX\_single(pi\_inv, G, MCMCsampleP, MCMCsampleW, MAPestP, MAPestW)

#### Arguments

pi_inv	An object of class top_ordering, collecting the numeric $N \times K$ data matrix of partial orderings, or an object that can be coerced with as.top_ordering.
G	Number of mixture components.
MCMCsampleP	Numeric $L \times G * K$ matrix with the MCMC samples of the component-specific support parameters to be processed.
MCMCsampleW	Numeric $L \times G$ matrix with the MCMC samples of the mixture weights to be processed.
MAPestP	Numeric $G \times K$ matrix of MAP component-specific support parameter estimates to be used as pivot in the PRA method.
MAPestW	Numeric vector of the $G$ MAP estimates of the mixture weights as pivot in the PRA method.

### Details

The label\_switchPLMIX function performs the label switching adjustment of the MCMC samples via the Pivotal Reordering Algorithm (PRA) described in Marin et al (2005), by recalling the pra function from the label.switching package.

# Value

A list of named objects:

final_sampleP	Numeric $G \times K \times L$ array MCMC samples of the component-specific support parameters adjusted for label switching.
final_sampleW	Numeric $L \times G$ matrix of MCMC samples of the mixture weights adjusted for label switching.

# Author(s)

Cristina Mollica and Luca Tardella

Loglikelihood	Likelihood and log-likelihood evaluation for a mixture of Plackett-
	Luce models

# Description

Compute either the likelihood or the log-likelihood of the Plackett-Luce mixture model parameters for a partial ordering dataset.

# Usage

likPLMIX(p, ref\_order, weights, pi\_inv)

loglikPLMIX(p, ref\_order, weights, pi\_inv)

### Arguments

р	Numeric $G \times K$ matrix of component-specific support parameters.
ref_order	Numeric $G \times K$ matrix of component-specific reference orders.
weights	Numeric vector of G mixture weights.
pi_inv	An object of class top_ordering, collecting the numeric $N \times K$ data matrix of partial orderings, or an object that can be coerced with as.top_ordering.

### Details

The ref\_order argument accommodates for the more general mixture of Extended Plackett-Luce models (EPL), involving the additional reference order parameters (Mollica and Tardella 2014). A permutation of the first K integers can be specified in each row of the ref\_order argument. Since the Plackett-Luce model is a special instance of the EPL with the reference order equal to the identity permutation, the ref\_order argument must be a matrix with G rows equal to  $(1, \ldots, K)$  when dealing with Plackett-Luce mixtures.

#### make\_complete

### Value

Either the likelihood or the log-likelihood value of the Plackett-Luce mixture model parameters for a partial ordering dataset.

# Author(s)

Cristina Mollica and Luca Tardella

# References

Mollica, C. and Tardella, L. (2017). Bayesian Plackett-Luce mixture models for partially ranked data. *Psychometrika*, **82**(2), pages 442–458, ISSN: 0033-3123, <doi:10.1007/s11336-016-9530-0>.

Mollica, C. and Tardella, L. (2014). Epitope profiling via mixture modeling for ranked data. *Statistics in Medicine*, **33**(21), pages 3738–3758, ISSN: 0277-6715, <doi:10.1002/sim.6224>.

# Examples

make\_complete

Completion of partial rankings/orderings

### Description

Return complete rankings/orderings from partial sequences relying on a random generation of the missing positions/items.

#### Usage

```
make_complete(
   data,
   format_input,
   nranked = NULL,
   probitems = rep(1, ncol(data))
)
```

### Arguments

data	Numeric $N \times K$ data matrix of partial sequences to be completed.
format_input	Character string indicating the format of the data input, namely "ordering" or "ranking".
nranked	Optional numeric vector of length $N$ with the number of items ranked by each sample unit.
probitems	Numeric vector with the $K$ item-specific probabilities to be employed for the random generation of the missing positions/items (see 'Details'). Default is equal probabilities.

### Details

The completion of the partial top rankings/orderings is performed according to the Plackett-Luce scheme, that is, with a sampling without replacement of the not-ranked items by using the positive values in the probitems argument as support parameters (normalization is not necessary).

### Value

A list of two named objects:

completedata	Numeric $N \times K$ data matrix of complete sequences with the same format of the input data.
nranked	Numeric vector of length $N$ with the number of items ranked by each sample unit of the input data.

### Author(s)

Cristina Mollica and Luca Tardella

make\_partial

#### Description

Return partial top rankings/orderings from complete sequences obtained either with user-specified censoring patterns or with a random truncation.

### Usage

```
make_partial(
   data,
   format_input,
   nranked = NULL,
   probcens = rep(1, ncol(data) - 1)
)
```

#### Arguments

data	Numeric $N \times K$ data matrix of complete sequences to be censored.
format_input	Character string indicating the format of the data input, namely "ordering" or "ranking".
nranked	Numeric vector of length $N$ with the desired number of items ranked by each sample unit after censoring. If not supplied (NULL), the censoring patterns are randomly generated according to the probabilities in the probcens argument.
probcens	Numeric vector of length $(K-1)$ with the probability of each censoring pattern to be employed for the random truncation of the complete sequences (normaliza- tion is not necessary). It works only if nranked argument is NULL (see 'Details'). Default is equal probabilities.

### Details

The censoring of the complete sequences can be performed in: (i) a deterministic way, by specifying the number of top positions to be retained for each sample unit in the nranked argument; (ii) a random way, by sequentially specifying the probabilities of the top-1, top-2, ..., top-(K - 1) censoring patterns in the probcens argument. Recall that a top-(K - 1) sequence corresponds to a complete ordering/ranking.

### Value

A list of two named objects:

partialdata	Numeric $N \times K$ data matrix of partial (censored) sequences with the same for- mat of the input data and missing positions/items denoted with zero entries.
nranked	Numeric vector of length $N$ with the number of items ranked by each sample unit after censoring.

### Author(s)

Cristina Mollica and Luca Tardella

#### Examples

## Check consistency with the nominal censoring probabilities round(prop.table(table(d\_german\_cens\$nranked)), 2)

mapPLMIX

MAP estimation for a Bayesian mixture of Plackett-Luce models

## Description

Perform MAP estimation via EM algorithm for a Bayesian mixture of Plackett-Luce models fitted to partial orderings.

# Usage

```
mapPLMIX(
    pi_inv,
    K,
    G,
    init = list(p = NULL, omega = NULL),
    n_iter = 1000,
    hyper = list(shape0 = matrix(1, nrow = G, ncol = K), rate0 = rep(0, G), alpha0 = rep(1,
    G)),
    eps = 10^(-6),
    centered_start = FALSE,
    plot_objective = FALSE
)
```

# Arguments

pi_inv	An object of class top_ordering, collecting the numeric $N \times K$ data matrix of partial orderings, or an object that can be coerced with as.top_ordering.
К	Number of possible items.
G	Number of mixture components.
init	List of named objects with initialization values: p is a numeric $G \times K$ matrix of component-specific support parameters; omega is a numeric vector of $G$ mixture weights. If starting values are not supplied (NULL), they are randomly generated with a uniform distribution. Default is NULL.

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n_iter	Maximum number of EM iterations.
hyper	List of named objects with hyperparameter values for the conjugate prior spec- ification: shape0 is a numeric $G \times K$ matrix of shape hyperparameters; rate0 is a numeric vector of G rate hyperparameters; alpha0 is a numeric vector of G Dirichlet hyperparameters. Default is noninformative (flat) prior setting.
eps	Tolerance value for the convergence criterion.
centered_start	Logical: whether a random start whose support parameters and weights should be centered around the observed relative frequency that each item has been ranked top. Default is FALSE. Ignored when init is not NULL.
<pre>plot_objective</pre>	Logical: whether the objective function (that is the kernel of the log-posterior distribution) should be plotted. Default is FALSE.

# Details

Under noninformative (flat) prior setting, the EM algorithm for MAP estimation corresponds to the EMM algorithm described by Gormley and Murphy (2006) to perform frequentist inference. In this case, the MAP solution coincides with the MLE and the output vectors log\_lik and objective coincide as well.

The mapPLMIX function performs the MAP procedure with a single starting value. To address the issue of local maxima in the posterior distribution, see the mapPLMIX\_multistart function.

### Value

A list of S3 class mpPLMIX with named elements:

W_map	Numeric vector with the MAP estimates of the $G$ mixture weights.
P_map	Numeric $G \times K$ matrix with the MAP estimates of the component-specific support parameters.
z_hat	Numeric $N \times G$ matrix of estimated posterior component membership probabil- ities.
class_map	Numeric vector of $N$ mixture component memberships based on MAP allocation from the z_hat matrix.
log_lik	Numeric vector of the log-likelihood values at each iteration.
objective	Numeric vector of the objective function values (that is the kernel of the log- posterior distribution) at each iteration.
<pre>max_objective</pre>	Maximized objective function value.
bic	BIC value (only for the default flat priors, otherwise NULL).
conv	Binary convergence indicator: $1 = \text{convergence has been achieved}, 0 = \text{otherwise}.$
call	The matched call.

# Author(s)

Cristina Mollica and Luca Tardella

### References

Mollica, C. and Tardella, L. (2017). Bayesian Plackett-Luce mixture models for partially ranked data. *Psychometrika*, **82**(2), pages 442–458, ISSN: 0033-3123, <doi:10.1007/s11336-016-9530-0>.

Gormley, I. C. and Murphy, T. B. (2006). Analysis of Irish third-level college applications data. *Journal of the Royal Statistical Society: Series A*, **169**(2), pages 361–379, ISSN: 0964-1998, <doi:10.1111/j.1467-985X.2006.00412.x>.

### See Also

mapPLMIX\_multistart

### Examples

```
data(d_carconf)
MAP <- mapPLMIX(pi_inv=d_carconf, K=ncol(d_carconf), G=3, n_iter=400*3)
str(MAP)
MAP$P_map
MAP$W_map</pre>
```

<pre>mapPLMIX_multistart</pre>	MAP estimation for a Bayesian mixture of Plackett-Luce models with
	multiple starting values

### Description

Perform MAP estimation via EM algorithm with multiple starting values for a Bayesian mixture of Plackett-Luce models fitted to partial orderings.

### Usage

```
mapPLMIX_multistart(
    pi_inv,
    K,
    G,
    n_start = 1,
    init = rep(list(list(p = NULL, omega = NULL)), times = n_start),
    n_iter = 200,
    hyper = list(shape0 = matrix(1, nrow = G, ncol = K), rate0 = rep(0, G), alpha0 = rep(1,
    G)),
    eps = 10^(-6),
    plot_objective = FALSE,
    init_index = 1:n_start,
    parallel = FALSE,
    centered_start = FALSE
)
```

# Arguments

pi_inv	An object of class top_ordering, collecting the numeric $N \times K$ data matrix of partial orderings, or an object that can be coerced with as.top_ordering.
К	Number of possible items.
G	Number of mixture components.
n_start	Number of starting values.
init	List of n_start lists of named objects with initialization values: p is a numeric $G \times K$ matrix of component-specific support parameters; omega is a numeric vector of $G$ mixture weights. If starting values are not supplied (NULL), they are randomly generated with a uniform distribution. Default is NULL.
n_iter	Maximum number of EM iterations.
hyper	List of named objects with hyperparameter values for the conjugate prior spec- ification: shape0 is a numeric $G \times K$ matrix of shape hyperparameters; rate0 is a numeric vector of G rate hyperparameters; alpha0 is a numeric vector of G Dirichlet hyperparameters. Default is noninformative (flat) prior setting.
eps	Tolerance value for the convergence criterion.
<pre>plot_objective</pre>	Logical: whether the objective function (that is the kernel of the log-posterior distribution) should be plotted. Default is FALSE.
init_index	Numeric vector indicating the positions of the starting values in the init list to be actually launched. Useful to launch the most promising starting values identified after a preliminary run. Default is run all the starting points in the init list.
parallel	Logical: whether parallelization should be used. Default is FALSE.
centered_start	Logical: whether a random start whose support parameters and weights should be centered around the observed relative frequency that each item has been ranked top. Default is FALSE. Ignored when init is not NULL.

# Details

Under noninformative (flat) prior setting, the EM algorithm for MAP estimation corresponds to the EMM algorithm described by Gormley and Murphy (2006) to perform frequentist inference. In this case the MAP solution coincides with the MLE. The best model in terms of maximized posterior distribution is returned.

# Value

A list of S3 class mpPLMIX with named elements:

mod	List of named objects describing the best model in terms of maximized posterior distribution. See output values of the single-run mapPLMIX function for a detailed explanation of the list elements.
<pre>max_objective</pre>	Numeric vector of the maximized objective function values for each initializa- tion.
convergence	Binary vector with length(init_index) convergence indicators for each ini- tialization: 1 = convergence has been achieved, 0 = otherwise.
call	The matched call.

### Author(s)

Cristina Mollica and Luca Tardella

#### References

Mollica, C. and Tardella, L. (2017). Bayesian Plackett-Luce mixture models for partially ranked data. *Psychometrika*, **82**(2), pages 442–458, ISSN: 0033-3123, <doi:10.1007/s11336-016-9530-0>.

Gormley, I. C. and Murphy, T. B. (2006). Analysis of Irish third-level college applications data. *Journal of the Royal Statistical Society: Series A*, **169**(2), pages 361–379, ISSN: 0964-1998, <doi:10.1111/j.1467-985X.2006.00412.x>.

#### See Also

mapPLMIX

#### Examples

myorder	Utility to switch from a partial ranking to a partial ordering (missing
	positions denoted with zero)

# Description

Utility to switch from a partial ranking to a partial ordering (missing positions denoted with zero)

#### Usage

myorder(x)

#### Arguments

Numeric integer vector

#### Author(s)

х

Cristina Mollica and Luca Tardella

paired\_comparisons Paired comparison matrix for a partial ordering/ranking dataset

# Description

Construct the paired comparison matrix for a partial ordering/ranking dataset.

### Usage

```
paired_comparisons(data, format_input, nranked = NULL)
```

### Arguments

data	Numeric $N \times K$ data matrix of partial sequences.
format_input	Character string indicating the format of the data input, namely "ordering" or "ranking".
nranked	Optional numeric vector of length ${\cal N}$ with the number of items ranked by each sample unit.

# Value

Numeric  $K \times K$  paired comparison matrix: the (i, i')-th entry indicates the number of sample units that preferred item i to item i'.

# Author(s)

Cristina Mollica and Luca Tardella

### References

Mollica, C. and Tardella, L. (2017). Bayesian Plackett-Luce mixture models for partially ranked data. *Psychometrika*, **82**(2), pages 442–458, ISSN: 0033-3123, <doi:10.1007/s11336-016-9530-0>.

#### See Also

rank\_summaries

```
data(d_dublinwest)
paired_comparisons(data=d_dublinwest, format_input="ordering")
```

PLMIX

PLMIX

Bayesian Analysis of Finite Mixtures of Plackett-Luce Models for Partial Rankings/Orderings

### Description

The **PLMIX** package for R provides functions to fit and analyze finite mixtures of Plackett-Luce models for partial top rankings/orderings within the Bayesian framework. It provides MAP point estimates via EM algorithm and posterior MCMC simulations via Gibbs Sampling. It also fits MLE as a special case of the noninformative Bayesian analysis with vague priors.

#### Details

In addition to inferential techniques, the package assists other fundamental phases of a model-based analysis for partial rankings/orderings, by including functions for data manipulation, simulation, descriptive summary, model selection and goodness-of-fit evaluation.

Specific S3 classes and methods are also supplied to enhance the usability and foster exchange with other packages. Finally, to address the issue of computationally demanding procedures typical in ranking data analysis, **PLMIX** takes advantage of a hybrid code linking the R environment with the C++ programming language.

The Plackett-Luce model is one of the most popular and frequently applied parametric distributions to analyze partial top rankings/orderings of a finite set of items. The present package allows to account for unobserved sample heterogeneity of partially ranked data with a model-based analysis relying on Bayesian finite mixtures of Plackett-Luce models. The package provides a suite of functions that covers the fundamental phases of a model-based analysis:

#### **Ranking data manipulation**

binary\_group\_ind Binary group membership matrix from the mixture component labels.

freq\_to\_unit From the frequency distribution to the dataset of individual orderings/rankings.

make\_complete Random completion of partial orderings/rankings data.

make\_partial Censoring of complete orderings/rankings data.

rank\_ord\_switch From rankings to orderings and vice-versa.

unit\_to\_freq From the dataset of individual orderings/rankings to the frequency distribution.

#### **Ranking data simulation**

rPLMIX Random sample from a finite mixture of Plackett-Luce models.

#### **Ranking data description**

paired\_comparisons Paired comparison frequencies.

rank\_summaries Summary statistics of partial ranking/ordering data.

#### Model estimation

gibbsPLMIX Bayesian analysis with MCMC posterior simulation via Gibbs sampling.

### PLMIX

label\_switchPLMIX Label switching adjustment of the Gibbs sampling simulations. likPLMIX Likelihood evaluation for a mixture of Plackett-Luce models. loglikPLMIX Log-likelihood evaluation for a mixture of Plackett-Luce models. mapPLMIX MAP estimation via EM algorithm.

 $mapPLMIX\_multistart$  MAP estimation via EM algorithm with multiple starting values.

### **Class coercion and membership**

as.top\_ordering Coercion into top-ordering datasets. gsPLMIX\_to\_mcmc From the Gibbs sampling simulation to an MCMC class object. is.top\_ordering Test for the consistency of input data with a top-ordering dataset.

#### S3 class methods

plot.gsPLMIX Plot of the Gibbs sampling simulations. plot.mpPLMIX Plot of the MAP estimates. print.gsPLMIX Print of the Gibbs sampling simulations. print.mpPLMIX Print of the MAP estimation algorithm. summary.gsPLMIX Summary of the Gibbs sampling procedure. summary.mpPLMIX Summary of the MAP estimation.

### Model selection

bicPLMIX BIC value for the MLE of a mixture of Plackett-Luce models. selectPLMIX Bayesian model selection criteria.

#### Model assessment

ppcheckPLMIX Posterior predictive diagnostics.

ppcheckPLMIX\_cond Posterior predictive diagnostics conditionally on the number of ranked items.

#### Datasets

d\_apa American Psychological Association Data (partial orderings).

d\_carconf Car Configurator Data (partial orderings).

d\_dublinwest Dublin West Data (partial orderings).

d\_gaming Gaming Platforms Data (complete orderings).

d\_german German Sample Data (complete orderings).

d\_nascar NASCAR Data (partial orderings).

d\_occup Occupation Data (complete orderings).

d\_rice Rice Voting Data (partial orderings).

Data have to be supplied as an object of class matrix, where missing positions/items are denoted with zero entries and Rank = 1 indicates the most-liked alternative. For a more efficient implementation of the methods, partial sequences with a single missing entry should be preliminarily filled in, as they correspond to complete rankings/orderings. In the present setting, ties are not allowed. Some quantities frequently recalled in the manual are the following:

- N Sample size.
- K Number of possible items.
- G Number of mixture components.
- L Size of the final posterior MCMC sample (after burn-in phase).

#### Author(s)

Cristina Mollica and Luca Tardella

Maintainer: Cristina Mollica <cristina.mollica@uniroma1.it>

#### References

Mollica, C. and Tardella, L. (2017). Bayesian Plackett-Luce mixture models for partially ranked data. *Psychometrika*, **82**(2), pages 442–458, ISSN: 0033-3123, <doi:10.1007/s11336-016-9530-0>.

Mollica, C. and Tardella, L. (2014). Epitope profiling via mixture modeling for ranked data. *Statistics in Medicine*, **33**(21), pages 3738–3758, ISSN: 0277-6715, <doi:10.1002/sim.6224/full>.

plot.gsPLMIX	Plot the Gibbs sampling simulations for a Bayes	ian mixture of
	Plackett-Luce models	

# Description

plot method for class gsPLMIX. It builds a suite of plots, visual convergence diagnostics and credible intervals for the MCMC samples of a Bayesian mixture of Plackett-Luce models. Graphics can be plotted directly into the current working device or stored into an external file placed into the current working directory.

#### Usage

```
## S3 method for class 'gsPLMIX'
plot(
    x,
    file = "ggmcmc-output.pdf",
    family = NA,
    plot = NULL,
    param_page = 5,
    width = 7,
    height = 10,
    dev_type_html = "png",
    post_est = "mean",
    max_scale_radar = NULL,
    ...
)
```

### Arguments

x	Object of class gsPLMIX returned by the gibbsPLMIX function.	
file	Character vector with the name of the file to be created in the current working directory. Defaults is "ggmcmc-output.pdf". When NULL, plots are directly returned into the current working device (not recommended). This option allows also the user to work with an opened pdf (or other) device. When the file has an html file extension, the output is an Rmarkdown report with the figures embedded in the html file.	
family	Character string indicating the name of the family of parameters to be plotted. A family of parameters is considered to be any group of parameters with the same name but different numerical values (for example w[1], w[2], etc). Default is NA meaning that all the parameters in the chain are plotted. Alternatively, one can choose "w", "p", "log_lik", "deviance" or "objective".	
plot	Character vector containing the names of the desired plots. Default is NULL meaning that all the plots and convergence diagnostics are built (see 'Details').	
param_page	Number of parameters to be plotted in each page. Defaults is 5.	
width	Numeric scalar indicating the width of the pdf display in inches. Defaults is 7.	
height	Numeric scalar indicating the height of the pdf display in inches. Defaults is 10.	
dev_type_html	Character vector indicating the type of graphical device for the html output. Default is "png". Alternatively, one can choose "svg".	
post_est	Character string indicating the point estimates of the Plackett-Luce mixture parameters to be computed from the gsPLMIX class object and then plotted in the current working device. Default is "mean". Alternatively, one can choose "median".	
max_scale_radar		
	Numeric scalar indicating the maximum value on each axis of the radar plot for the support parameter point estimates. Default is NULL meaning that the maximum of the estimated support parameters is used.	
	Further arguments passed to or from other methods (not used).	

# Details

Plots of the MCMC samples include histograms, densities, traceplots, running means plots, overlapped densities comparing the complete and partial samples, autocorrelation functions, crosscorrelation plots and caterpillar plots of the 90 and 95% equal-tails credible intervals. Note that the latter are created for the support parameters (when either family=NA or family="p"), for the mixture weights in the case G > 1 (when either family=NA or family="w"), for the log-likelihood values (when family="log\_lik"), for the deviance values (when family="deviance"). Convergence tools include the potential scale reduction factor and the Geweke z-score. These functionalities are implemented with a call to the ggs and ggmcmc functions of the ggmcmc package (see 'Examples' for the specification of the plot argument) and for the objective function values (when family="objective").

By recalling the chartJSRadar function from the radarchart package and the routines of the ggplot2 package, plot.gsPLMIX additionally produces a radar plot of the support parameters and, when G > 1, a donut plot of the mixture weights based on the posterior point estimates. The radar chart is returned in the Viewer Pane.

#### Author(s)

Cristina Mollica and Luca Tardella

#### References

Ashton, D. and Porter, S. (2016). radarchart: Radar Chart from 'Chart.js'. R package version 0.3.1. https://CRAN.R-project.org/package=radarchart

Wickham, H. (2009). ggplot2: Elegant Graphics for Data Analysis. Springer-Verlag New York.

Fernandez-i-Marin, X. (2006). ggmcmc: Analysis of MCMC Samples and Bayesian Inference, *Journal of Statistical Software*, **70**(9), pages 1–20, <doi:10.18637/jss.v070.i09>.

### See Also

ggs, ggmcmc, chartJSRadar and ggplot

#### Examples

```
# Not run:
data(d_carconf)
GIBBS <- gibbsPLMIX(pi_inv=d_carconf, K=ncol(d_carconf), G=5, n_iter=30, n_burn=10)
# Not run:
# Plot posterior samples supplied as an gsPLMIX class object
# plot(GIBBS)
# Selected plots of the posterior samples of the support parameters
# plot(GIBBS, family="p", plot=c("compare_partial","Rhat","caterpillar"), param_page=6)
# Selected plots of the posterior samples of the mixture weights
# plot(GIBBS, family="w", plot=c("histogram","running","crosscorrelation","caterpillar"))
# Selected plots of the posterior log-likelihood values
# plot(GIBBS, family="log_lik", plot=c("autocorrelation","geweke"), param_page=1)
# Selected plots of the posterior deviance values
# plot(GIBBS, family="deviance", plot=c("traceplot","density"), param_page=1)
```

plot.mpPLMIX	Plot the MAP estimates for a Bayesian mixture of Plackett-Luce mod-
	els

### Description

plot method for class mpPLMIX.

#### Usage

## S3 method for class 'mpPLMIX'
plot(x, max\_scale\_radar = NULL, ...)
# plot.mpPLMIX

## Arguments

х

Object of class mpPLMIX returned by the mpPLMIX function.

<pre>max_scale_rada</pre>	r
	Numeric scalar indicating the maximum value on each axis of the radar plot for the support parameter point estimates. Default is NULL meaning that the maximum of the estimated support parameters is used.
	Further arguments passed to or from other methods (not used).

# Details

By recalling the chartJSRadar function from the radarchart package and the routines of the ggplot2 package, plot.mpPLMIX produces a radar plot of the support parameters and, when G > 1, a donut plot of the mixture weights and a heatmap of the component membership probabilities based on the MAP estimates. The radar chart is returned in the Viewer Pane.

## Author(s)

Cristina Mollica and Luca Tardella

# References

Ashton, D. and Porter, S. (2016). radarchart: Radar Chart from 'Chart.js'. R package version 0.3.1. https://CRAN.R-project.org/package=radarchart

Wickham, H. (2009). ggplot2: Elegant Graphics for Data Analysis. Springer-Verlag New York.

#### See Also

chartJSRadar and ggplot

```
# Not run:
data(d_carconf)
MAP <- mapPLMIX(pi_inv=d_carconf, K=ncol(d_carconf), G=3)
plot(MAP)
# Not run:
```

```
MAP_multi <- mapPLMIX_multistart(pi_inv=d_carconf, K=ncol(d_carconf), G=3, n_start=5)
plot(MAP_multi)</pre>
```

ppcheckPLMIX

# Description

Perform posterior predictive check to assess the goodness-of-fit of Bayesian mixtures of Plackett-Luce models with a different number of components.

## Usage

```
ppcheckPLMIX(
    pi_inv,
    seq_G,
    MCMCsampleP,
    MCMCsampleW,
    top1 = TRUE,
    paired = TRUE,
    parallel = FALSE
)
```

## Arguments

pi_inv	An object of class top_ordering, collecting the numeric $N \times K$ data matrix of partial orderings, or an object that can be coerced with as.top_ordering.
seq_G	Numeric vector with the number of components of the Plackett-Luce mixtures to be assessed.
MCMCsampleP	List of size length(seq_G), whose generic element is a numeric $L \times (G * K)$ matrix with the MCMC samples of the component-specific support parameters.
MCMCsampleW	List of size length(seq_G), whose generic element is a numeric $L \times G$ matrix with the MCMC samples of the mixture weights.
top1	Logical: whether the posterior predictive <i>p</i> -value based on the top item frequencies has to be computed. Default is TRUE.
paired	Logical: whether the posterior predictive <i>p</i> -value based on the paired comparison frequencies has to be computed. Default is TRUE.
parallel	Logical: whether parallelization should be used. Default is FALSE.

# Details

The ppcheckPLMIX function returns two posterior predictive *p*-values based on two chi squared discrepancy variables involving: (i) the top item frequencies and (ii) the paired comparison frequencies. In the presence of partial sequences in the pi\_inv matrix, the same missingness patterns observed in the dataset (i.e., the number of items ranked by each sample unit) are reproduced on the replicated datasets from the posterior predictive distribution.

## ppcheckPLMIX

#### Value

A list with a named element:

#### post\_pred\_pvalue

Numeric length(seq\_G) $\times 2$  matrix of posterior predictive *p*-values based on the top item and paired comparison frequencies. If either top1 or paired argument is FALSE, the corresponding matrix entries are NA.

# Author(s)

Cristina Mollica and Luca Tardella

#### References

Mollica, C. and Tardella, L. (2017). Bayesian Plackett-Luce mixture models for partially ranked data. *Psychometrika*, **82**(2), pages 442–458, ISSN: 0033-3123, <doi:10.1007/s11336-016-9530-0>.

# See Also

ppcheckPLMIX\_cond

```
data(d_carconf)
K <- ncol(d_carconf)</pre>
## Fit 1- and 2-component PL mixtures via MAP estimation
MAP_1 <- mapPLMIX_multistart(pi_inv=d_carconf, K=K, G=1,</pre>
                              n_start=2, n_iter=400*1)
MAP_2 <- mapPLMIX_multistart(pi_inv=d_carconf, K=K, G=2,</pre>
                               n_start=2, n_iter=400*2)
MAP_3 <- mapPLMIX_multistart(pi_inv=d_carconf, K=K, G=3,</pre>
                              n_start=2, n_iter=400*3)
mcmc_iter <- 30</pre>
burnin <- 10
## Fit 1- and 2-component PL mixtures via Gibbs sampling procedure
GIBBS_1 <- gibbsPLMIX(pi_inv=d_carconf, K=K, G=1, n_iter=mcmc_iter,</pre>
                       n_burn=burnin, init=list(p=MAP_1$mod$P_map,
                       z=binary_group_ind(MAP_1$mod$class_map,G=1)))
GIBBS_2 <- gibbsPLMIX(pi_inv=d_carconf, K=K, G=2, n_iter=mcmc_iter,</pre>
                       n_burn=burnin, init=list(p=MAP_2$mod$P_map,
                       z=binary_group_ind(MAP_2$mod$class_map,G=2)))
GIBBS_3 <- gibbsPLMIX(pi_inv=d_carconf, K=K, G=3, n_iter=mcmc_iter,</pre>
                       n_burn=burnin, init=list(p=MAP_3$mod$P_map,
                       z=binary_group_ind(MAP_3$mod$class_map,G=3)))
```

<sup>##</sup> Checking goodness-of-fit of the estimated mixtures CHECK <- ppcheckPLMIX(pi\_inv=d\_carconf, seq\_G=1:3,</pre>

```
MCMCsampleP=list(GIBBS_1$P, GIBBS_2$P, GIBBS_3$P),
MCMCsampleW=list(GIBBS_1$W, GIBBS_2$W, GIBBS_3$W))
```

CHECK\$post\_pred\_pvalue

ppcheckPLMIX\_cond Conditional posterior predictive check for Bayesian mixtures of Plackett-Luce models

# Description

Perform conditional posterior predictive check to assess the goodness-of-fit of Bayesian mixtures of Plackett-Luce models with a different number of components.

## Usage

```
ppcheckPLMIX_cond(
    pi_inv,
    seq_G,
    MCMCsampleP,
    MCMCsampleW,
    top1 = TRUE,
    paired = TRUE,
    parallel = FALSE
)
```

# Arguments

pi_inv	An object of class top_ordering, collecting the numeric $N \times K$ data matrix of partial orderings, or an object that can be coerced with as.top_ordering.
seq_G	Numeric vector with the number of components of the Plackett-Luce mixtures to be assessed.
MCMCsampleP	List of size length(seq_G), whose generic element is a numeric $L \times (G * K)$ matrix with the MCMC samples of the component-specific support parameters.
MCMCsampleW	List of size length(seq_G), whose generic element is a numeric $L \times G$ matrix with the MCMC samples of the mixture weights.
top1	Logical: whether the posterior predictive $p$ -value based on the top item frequencies has to be computed. Default is TRUE.
paired	Logical: whether the posterior predictive $p$ -value based on the paired comparison frequencies has to be computed. Default is TRUE.
parallel	Logical: whether parallelization should be used. Default is FALSE.

## Details

The ppcheckPLMIX\_cond function returns two posterior predictive *p*-values based on two chi squared discrepancy variables involving: (i) the top item frequencies and (ii) the paired comparison frequencies. In the presence of partial sequences in the pi\_inv matrix, the same missingness patterns observed in the dataset (i.e., the number of items ranked by each sample unit) are reproduced on the replicated datasets from the posterior predictive distribution. Differently from the ppcheckPLMIX function, the condional discrepancy measures are obtained by summing up the chi squared discrepancies computed on subsamples of observations with the same number of ranked items.

## Value

A list with a named element:

post\_pred\_pvalue\_cond

Numeric length(seq\_G) $\times 2$  matrix of posterior predictive *p*-values based on the top item and paired comparison frequencies. If either top1 or paired argument is FALSE, the corresponding matrix entries are NA.

# Author(s)

Cristina Mollica and Luca Tardella

## References

Mollica, C. and Tardella, L. (2017). Bayesian Plackett-Luce mixture models for partially ranked data. *Psychometrika*, **82**(2), pages 442–458, ISSN: 0033-3123, <doi:10.1007/s11336-016-9530-0>.

## See Also

#### ppcheckPLMIX

```
n_burn=burnin, init=list(p=MAP_1$mod$P_map,
z=binary_group_ind(MAP_1$mod$class_map,G=1)))
GIBBS_2 <- gibbsPLMIX(pi_inv=d_carconf, K=K, G=2, n_iter=mcmc_iter,
n_burn=burnin, init=list(p=MAP_2$mod$P_map,
z=binary_group_ind(MAP_2$mod$class_map,G=2)))
GIBBS_3 <- gibbsPLMIX(pi_inv=d_carconf, K=K, G=3, n_iter=mcmc_iter,
n_burn=burnin, init=list(p=MAP_3$mod$P_map,
z=binary_group_ind(MAP_3$mod$class_map,G=3)))
## Checking goodness-of-fit of the estimated mixtures
CHECKCOND <- ppcheckPLMIX_cond(pi_inv=d_carconf, seq_G=1:3,
MCMCsampleP=list(GIBBS_1$P, GIBBS_2$P, GIBBS_3$P),
MCMCsampleW=list(GIBBS_1$W, GIBBS_2$W, GIBBS_3$W))
CHECKCOND$post_pred_pvalue
```

ppcheckPLMIX\_cond\_single

Conditional predictive posterior p-values

## Description

Compute conditional predictive posterior *p*-values based on top paired comparison frequencies to assess the goodness-of-fit of a Bayesian mixtures of Plackett-Luce models for partial orderings.

# Usage

```
ppcheckPLMIX_cond_single(
    pi_inv,
    G,
    MCMCsampleP,
    MCMCsampleW,
    top1 = TRUE,
    paired = TRUE
)
```

## Arguments

pi_inv	An object of class top_ordering, collecting the numeric $N \times K$ data matrix of partial orderings, or an object that can be coerced with as.top_ordering.
G	Number of mixture components.
MCMCsampleP	Numeric $L \times G * K$ matrix with the MCMC samples of the component-specific support parameters.
MCMCsampleW	Numeric $L \times G$ matrix with the MCMC samples of the mixture weights.
top1	Logical: whether the posterior predictive <i>p</i> -value based on top frequencies has to be computed. Default is TRUE.
paired	Logical: whether the posterior predictive <i>p</i> -value based on paired comparison frequencies has to be computed. Default is TRUE.

# Details

In the case of partial orderings, the same missingness patterns of the observed dataset, i.e., the number of items ranked by each sample unit, are reproduced on the replicated datasets.

# Value

A list of named objects:

post\_pred\_pvalue\_top1 If top1 is TRUE, posterior predictive p-value based on top frequencies, otherwise NULL. post\_pred\_pvalue\_paired If paired is TRUE, posterior predictive p-value based on paired comparison frequencies, otherwise NULL.

## Author(s)

Cristina Mollica and Luca Tardella

ppcheckPLMIX\_single Posterior predictive check for a mixture of Plackett-Luce models

## Description

Compute predictive posterior p-values based on top item and paired comparison frequencies to assess the goodness-of-fit of a Bayesian mixtures of Plackett-Luce models for partial orderings.

## Usage

```
ppcheckPLMIX_single(
 pi_inv,
 G,
 MCMCsampleP,
 MCMCsampleW,
 top1 = TRUE,
  paired = TRUE
)
```

## Arguments

pi_inv	An object of class top_ordering, collecting the numeric $N \times K$ data matrix of partial orderings, or an object that can be coerced with as.top_ordering.
G	Number of mixture components.
MCMCsampleP	Numeric $L \times G * K$ matrix with the MCMC samples of the component-specific support parameters.
MCMCsampleW	Numeric $L \times G$ matrix with the MCMC samples of the mixture weights.

top1	Logical: whether the posterior predictive <i>p</i> -value based on top frequencies has to be computed. Default is TRUE.
paired	Logical: whether the posterior predictive <i>p</i> -value based on paired comparison frequencies has to be computed. Default is TRUE.

# Details

In the case of partial orderings, the same missingness patterns of the observed dataset, i.e., the number of items ranked by each sample unit, are reproduced on the replicated datasets.

# Value

A list of named objects:

## Author(s)

Cristina Mollica and Luca Tardella

print.gsPLMIX	Print of the Gibbs sampling simulation of a Bayesian mixture of
	Plackett-Luce models

# Description

print method for class gsPLMIX. It shows some general information on the Gibbs sampling simulation for a Bayesian mixture of Plackett-Luce models.

## Usage

```
## S3 method for class 'gsPLMIX'
print(x, ...)
```

#### Arguments

х	Object of class gsPLMIX returned by the gibbsPLMIX function.
	Further arguments passed to or from other methods (not used).

# Author(s)

Cristina Mollica and Luca Tardella

# print.mpPLMIX

## See Also

## gibbsPLMIX

## Examples

```
## Print of the Gibbs sampling procedure
data(d_carconf)
GIBBS <- gibbsPLMIX(pi_inv=d_carconf, K=ncol(d_carconf), G=3, n_iter=30, n_burn=10)
print(GIBBS)</pre>
```

print.mpPLMIX	Print of the MAP estimation algorithm for a Bayesian mixture of
	Plackett-Luce models

## Description

print method for class mpPLMIX. It shows some general information on the MAP estimation procedure for a Bayesian mixture of Plackett-Luce models.

# Usage

```
## S3 method for class 'mpPLMIX'
print(x, ...)
```

## Arguments

х	Object of class mpPLMIX returned by the mapPLMIX or mapPLMIX_multistart function.
•••	Further arguments passed to or from other methods (not used).

# Author(s)

Cristina Mollica and Luca Tardella

## See Also

mapPLMIX and mapPLMIX\_multistart

```
## Print of the MAP procedure with a single starting point
data(d_carconf)
MAP <- mapPLMIX(pi_inv=d_carconf, K=ncol(d_carconf), G=3)
print(MAP)
## Print of the MAP procedure with 5 starting points
```

```
MAP_multi <- mapPLMIX_multistart(pi_inv=d_carconf, K=ncol(d_carconf), G=3, n_start=5)
print(MAP_multi)</pre>
```

print.summary.gsPLMIX Print of the summary of Gibbs sampling simulation of a Bayesian mixture of Plackett-Luce models.

## Description

print method for class summary.gsPLMIX. It shows some general information on the Gibbs sampling simulation of a Bayesian mixture of Plackett-Luce models.

## Usage

```
## S3 method for class 'summary.gsPLMIX'
print(x, ...)
```

## Arguments

х	Object of class summary.gsPLMIX returned by the summary.gsPLMIX function.
	Further arguments passed to or from other methods (not used).

#### Author(s)

Cristina Mollica and Luca Tardella

# References

Mollica, C. and Tardella, L. (2017). Bayesian Plackett-Luce mixture models for partially ranked data. *Psychometrika*, **82**(2), pages 442–458, ISSN: 0033-3123, <doi:10.1007/s11336-016-9530-0>.

Mollica, C. and Tardella, L. (2014). Epitope profiling via mixture modeling for ranked data. *Statistics in Medicine*, **33**(21), pages 3738–3758, ISSN: 0277-6715, <doi:10.1002/sim.6224>.

print.summary.mpPLMIX Print of the summary of MAP estimation for a Bayesian mixture of Plackett-Luce models

## Description

print method for class summary.mpPLMIX. It provides summaries for the MAP estimation of a Bayesian mixture of Plackett-Luce models.

#### Usage

```
## S3 method for class 'summary.mpPLMIX'
print(x, ...)
```

## random\_start

#### Arguments

х	Object of class summary.mpPLMIX returned by the summary.mpPLMIX function
	Further arguments passed to or from other methods (not used).

## Author(s)

Cristina Mollica and Luca Tardella

## References

Mollica, C. and Tardella, L. (2017). Bayesian Plackett-Luce mixture models for partially ranked data. *Psychometrika*, **82**(2), pages 442–458, ISSN: 0033-3123, <doi:10.1007/s11336-016-9530-0>.

Mollica, C. and Tardella, L. (2014). Epitope profiling via mixture modeling for ranked data. *Statistics in Medicine*, **33**(21), pages 3738–3758, ISSN: 0277-6715, <doi:10.1002/sim.6224>.

random\_start

Appropriate simulation of starting values for tandom initialization of Gibbs Sampling. It start from the mle corresponding to no-group structure and then it randomly selects rescaled random support points (with sum 1) of G mixture components such that the marginal support coincides with the mle support for G=1 Random generation of starting values of the component-specific support parameters for Gibbs sampling

#### Description

Appropriate simulation of starting values for tandom initialization of Gibbs Sampling. It start from the mle corresponding to no-group structure and then it randomly selects rescaled random support points (with sum 1) of G mixture components such that the marginal support coincides with the mle support for G=1 Random generation of starting values of the component-specific support parameters for Gibbs sampling

#### Usage

```
random_start(mlesupp, givenweights, alpha = rep(1, G))
```

#### Arguments

mlesupp	MLE of support parameters
givenweights	A numeric vector of G mixture weights
alpha	A numeric vector of $G$ positive reals to be used as Dirichlet parameters for the random start which corresponds to a convex combination of $G$ support parameter vertices

## Value

out A numeric  $G \times K$  matrix with starting values of the component-specific support parameters

# Author(s)

Cristina Mollica and Luca Tardella

rank\_ord\_switch Switch from orderings to rankings and vice versa

# Description

Convert the format of the input dataset from orderings to rankings and vice versa.

# Usage

```
rank_ord_switch(data, format_input, nranked = NULL)
```

## Arguments

data	Numeric $N \times K$ data matrix of partial sequences whose format has to be converted.
format_input	Character string indicating the format of the data input, namely "ordering" or "ranking".
nranked	Optional numeric vector of length $N$ with the number of items ranked by each sample unit.

#### Value

Numeric  $N \times K$  data matrix of partial sequences with inverse format.

# Author(s)

Cristina Mollica and Luca Tardella

# Examples

```
## From orderings to rankings for the Dublin West dataset
data(d_dublinwest)
head(d_dublinwest)
rank_ord_switch(data=head(d_dublinwest), format_input="ordering")
```

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rank\_summaries

# Description

Compute rank summaries and censoring patterns for a partial ordering/ranking dataset.

# Usage

```
rank_summaries(
   data,
   format_input,
   mean_rank = TRUE,
   marginals = TRUE,
   pc = TRUE
)
```

# Arguments

data	Numeric $N \times K$ data matrix of partial sequences.
format_input	Character string indicating the format of the data input, namely "ordering" or "ranking".
mean_rank	Logical: whether the mean rank vector has to be computed. Default is TRUE.
marginals	Logical: whether the marginal rank distributions have to be computed. Default is TRUE.
рс	Logical: whether the paired comparison matrix has to be computed. Default is TRUE.

# Value

A list of named objects:

nranked	Numeric vector of length $N$ with the number of items ranked by each sample unit.
nranked_distr	Frequency distribution of the nranked vector.
na_or_not	Numeric $3 \times K$ matrix with the counts of sample units that ranked or not each item. The last row contains the total by column, corresponding to the sample size $N$ .
mean_rank	Numeric vector of length $K$ with the mean rank of each item.
marginals	Numeric $K \times K$ matrix of the marginal rank distributions: the $(i, j)$ -th entry indicates the number of units that ranked item $i$ in the $j$ -th position.
рс	Numeric $K \times K$ paired comparison matrix: the $(i, i')$ -th entry indicates the number of sample units that preferred item $i$ to item $i'$ .

## Author(s)

Cristina Mollica and Luca Tardella

## References

Marden, J. I. (1995). Analyzing and modeling rank data. *Monographs on Statistics and Applied Probability* (64). Chapman & Hall, ISSN: 0-412-99521-2. London.

# Examples

```
data(d_carconf)
rank_summaries(data=d_carconf, format_input="ordering")
```

rPLMIX

Random sample from a mixture of Plackett-Luce models

# Description

Draw a random sample of complete orderings/rankings from a *G*-component mixture of Plackett-Luce models.

## Usage

```
rPLMIX(
    n = 1,
    K,
    G,
    p = t(matrix(1/K, nrow = K, ncol = G)),
    ref_order = t(matrix(1:K, nrow = K, ncol = G)),
    weights = rep(1/G, G),
    format_output = "ordering"
)
```

```
Arguments
```

n	Number of observations to be sampled. Default is 1.
К	Number of possible items.
G	Number of mixture components.
р	Numeric $G \times K$ matrix of component-specific support parameters. Default is equal support parameters (uniform mixture components).
ref_order	Numeric $G \times K$ matrix of component-specific reference orders. Default is forward orders (identity permutations) in each row, corresponding to Plackett-Luce mixture components (see 'Details').
weights	Numeric vector of $G$ mixture weights. Default is equal weights.
format_output	Character string indicating the format of the returned simulated dataset ("ordering" or "ranking"). Default is "ordering".

## selectPLMIX

## Details

Positive values are required for p and weights arguments (normalization is not necessary).

The ref\_order argument accommodates for the more general mixture of Extended Plackett-Luce models (EPL), involving the additional reference order parameters (Mollica and Tardella 2014). A permutation of the first K integers can be specified in each row of the ref\_order argument to generate a sample from a G-component mixture of EPL. Since the Plackett-Luce model is a special instance of the EPL with the reference order equal to the identity permutation  $(1, \ldots, K)$ , the default value of the ref\_order argument is forward orders.

# Value

If G = 1, a numeric  $N \times K$  matrix of simulated complete sequences. If G > 1, a list of two named objects:

comp	Numeric vector of $N$ mixture component memberships.
sim_data	Numeric $N \times K$ matrix of simulated complete sequences.

## Author(s)

Cristina Mollica and Luca Tardella

#### Examples

```
K <- 6
G <- 3
support_par <- matrix(1:(G*K), nrow=G, ncol=K)
weights_par <- c(0.50, 0.25, 0.25)
set.seed(47201)
simulated_data <- rPLMIX(n=5, K=K, G=G, p=support_par, weights=weights_par)
simulated_data$comp
simulated_data$sim_data
```

selectPLMIX

Bayesian selection criteria for mixtures of Plackett-Luce models

# Description

Compute Bayesian comparison criteria for mixtures of Plackett-Luce models with a different number of components.

# Usage

```
selectPLMIX(
    pi_inv,
    seq_G,
    MCMCsampleP = vector(mode = "list", length = length(seq_G)),
    MCMCsampleW = vector(mode = "list", length = length(seq_G)),
    MAPestP,
    MAPestP,
    MAPestW,
    deviance,
    post_est = "mean",
    parallel = FALSE
)
```

# Arguments

pi_inv	An object of class top_ordering, collecting the numeric $N \times K$ data matrix of partial orderings, or an object that can be coerced with as.top_ordering.
seq_G	Numeric vector with the number of components of the Plackett-Luce mixtures to be compared.
MCMCsampleP	List of size length(seq_G), whose generic element is a numeric $L \times (G * K)$ matrix with the MCMC samples of the component-specific support parameters. Default is list of NULL elements.
MCMCsampleW	List of size length(seq_G), whose generic element is a numeric $L \times G$ matrix with the MCMC samples of the mixture weights. Default is list of NULL elements.
MAPestP	List of size length(seq_G), whose generic element is a numeric $G \times K$ matrix with the MAP estimates of the component-specific support parameters.
MAPestW	List of size length(seq_G), whose generic element is a numeric vector with the MAP estimates of the $G$ mixture weights.
deviance	List of size length(seq_G), whose generic element is a numeric vector of pos- terior deviance values.
post_est	Character string indicating the point estimates of the Plackett-Luce mixture parameters to be computed from the MCMC sample. This argument is ignored when MAP estimates are supplied in the MAPestP and MAPestW arguments. Default is "mean". Alternatively, one can choose "median" (see 'Details').
parallel	Logical: whether parallelization should be used. Default is FALSE.

## Details

The selectPLMIX function privileges the use of the MAP point estimates to compute the Bayesian model comparison criteria, since they are not affected by the label switching issue. By setting both the MAPestP and MAPestW arguments equal to NULL, the user can alternatively compute the selection measures by relying on a different posterior summary ("mean" or "median") specified in the post\_est argument. In the latter case, the MCMC samples for each Plackett-Luce mixture must be supplied in the lists MCMCsampleP and MCMCsampleW. The drawback when working with point estimates other than the MAP is that the possible presence of label switching has to be previously

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removed from the traces to obtain meaningful results. See the label\_switchPLMIX function to perfom label switching adjustment of the MCMC samples.

Several model selection criteria are returned. The two versions of DIC correspond to alternative ways of computing the effective number of parameters: DIC1 was proposed by Spiegelhalter et al. (2002) with penalty named pD, whereas DIC2 was proposed by Gelman et al. (2004) with penalty named pV. The latter coincides with the AICM introduced by Raftery et al. (2007), that is, the Bayesian counterpart of AIC. BPIC1 and BPIC2 are obtained from the two DIC by simply doubling the penalty term, as suggested by Ando (2007) to contrast DIC's tendency to overfitting. BICM1 is the Bayesian variant of the BIC, originally presented by Raftery et al. (2007) and entirely based on the MCMC sample. The BICM2, instead, involved the MAP estimate without the need of its approximation from the MCMC sample as for the BICM1.

## Value

A list of named objects:

point_estP	List of size length(seq_G), whose generic element is a numeric $G \times K$ matrix with the point estimates of the component-specific support parameters employed for the computation of the criteria.
point_estW	List of size length(seq_G), whose generic element is a numeric vector with the $G$ point estimates of the mixture weights employed for the computation of the criteria.
fitting	Numeric length(seq_G) $\times 2$ matrix with the fitting terms of the comparison measures, given by the posterior expected deviance D_bar and the deviance D_hat evaluated at the point estimate.
penalties	Numeric length(seq_G) $\times 2$ matrix with the penalty terms pD and pV (effective number of parameters).
criteria	Numeric length(seq_G) $\times$ 6 matrix of Bayesian model selection criteria: DIC1, DIC2, BPIC1, BPIC2, BICM1 and BICM2 (see 'Details').

## Author(s)

Cristina Mollica and Luca Tardella

#### References

Mollica, C. and Tardella, L. (2017). Bayesian Plackett-Luce mixture models for partially ranked data. *Psychometrika*, **82**(2), pages 442–458, ISSN: 0033-3123, <doi:10.1007/s11336-016-9530-0>.

Ando, T. (2007). Bayesian predictive information criterion for the evaluation of hierarchical Bayesian and empirical Bayes models. *Biometrika*, **94**(2), pages 443–458.

Raftery, A. E, Satagopan, J. M., Newton M. A. and Krivitsky, P. N. (2007). BAYESIAN STATIS-TICS 8. *Proceedings of the eighth Valencia International Meeting 2006*, pages 371–416. Oxford University Press.

Gelman, A., Carlin, J. B., Stern, H. S. and Rubin, D. B. (2004). Bayesian data analysis. Chapman & Hall/CRC, Second Edition, ISBN: 1-58488-388-X. New York.

Spiegelhalter, D. J., Best, N. G., Carlin, B. P. and Van Der Linde, A. (2002). Bayesian measures of model complexity and fit. *Journal of the Royal Statistical Society: Series B (Statistical Methodol-ogy)*, **64**(4), pages 583–639.

# Examples

```
data(d_carconf)
K <- ncol(d_carconf)</pre>
## Fit 1- and 2-component PL mixtures via MAP estimation
MAP_1 <- mapPLMIX_multistart(pi_inv=d_carconf, K=K, G=1,</pre>
                                     n_start=2, n_iter=400*1)
MAP_2 <- mapPLMIX_multistart(pi_inv=d_carconf, K=K, G=2,</pre>
                                     n_start=2, n_iter=400*2)
mcmc_iter <- 30</pre>
burnin <- 10
## Fit 1- and 2-component PL mixtures via Gibbs sampling procedure
GIBBS_1 <- gibbsPLMIX(pi_inv=d_carconf, K=K, G=1, n_iter=mcmc_iter,</pre>
                       n_burn=burnin, init=list(p=MAP_1$mod$P_map,
                       z=binary_group_ind(MAP_1$mod$class_map,G=1)))
GIBBS_2 <- gibbsPLMIX(pi_inv=d_carconf, K=K, G=2, n_iter=mcmc_iter,</pre>
                       n_burn=burnin, init=list(p=MAP_2$mod$P_map,
                       z=binary_group_ind(MAP_2$mod$class_map,G=2)))
## Select the optimal number of components
SELECT <- selectPLMIX(pi_inv=d_carconf, seq_G=1:2,</pre>
                       MAPestP=list(MAP_1$mod$P_map, MAP_2$mod$P_map),
                       MAPestW=list(MAP_1$mod$W_map, MAP_2$mod$W_map),
                       deviance=list(GIBBS_1$deviance, GIBBS_2$deviance))
SELECT$criteria
```

selectPLMIX\_single Bayesian selection criteria for mixtures of Plackett-Luce models

# Description

Compute Bayesian comparison criteria for mixtures of Plackett-Luce models with a different number of components.

## Usage

```
selectPLMIX_single(
    pi_inv,
    G,
    MCMCsampleP = NULL,
    MCMCsampleW = NULL,
    MAPestP,
    MAPestW,
    deviance,
    post_est = "mean"
)
```

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# Arguments

pi_inv	An object of class top_ordering, collecting the numeric $N \times K$ data matrix of partial orderings, or an object that can be coerced with as.top_ordering.
G	Number of mixture components.
MCMCsampleP	Numeric $L \times G * K$ matrix with the MCMC samples of the component-specific support parameters.
MCMCsampleW	Numeric $L \times G$ matrix with the MCMC samples of the mixture weights.
MAPestP	Numeric $G \times K$ matrix of MAP component-specific support parameter estimates.
MAPestW	Numeric vector of the G MAP estimates of the mixture weights.
deviance	Numeric vector of posterior deviance values.
post_est	Character string indicating the point estimates of the Plackett-Luce mixture parameters to be computed from the MCMC sample. This argument is ignored when MAP estimates are supplied in the MAPestP and MAPestW arguments. Default is "mean". Alternatively, one can choose "median".

# Details

Two versions of DIC and BPIC are returned corresponding to two alternative ways of computing the penalty term: the former was proposed by Spiegelhalter et al. (2002) and is denoted with pD, whereas the latter was proposed by Gelman et al. (2004) and is denoted with pV. DIC2 coincides with AICM, that is, the Bayesian counterpart of AIC introduced by Raftery et al. (2007).

#### Value

A list of named objects:

point_estP	Numeric $G \times (K + 1)$ matrix with the point estimates of the Plackett-Luce mixture parameters. The $(K + 1)$ -th column contains estimates of the mixture weights.
point_estW	Numeric $G \times (K + 1)$ matrix with the point estimates of the Plackett-Luce mixture parameters. The $(K + 1)$ -th column contains estimates of the mixture weights.
D_bar	Posterior expected deviance.
D_hat	Deviance function evaluated at point_est.
рD	Effective number of parameters computed as D_bar-D_hat.
рV	Effective number of parameters computed as half the posterior variance of the deviance.
DIC1	Deviance Information Criterion with penalty term equal to pD.
DIC2	Deviance Information Criterion with penalty term equal to pV.
BPIC1	Bayesian Predictive Information Criterion obtained from DIC1 by doubling its penalty term.
BPIC2	Bayesian Predictive Information Criterion obtained from DIC2 by doubling its penalty term.

BICM1	Bayesian Information Criterion-Monte Carlo.
BICM2	Bayesian Information Criterion-Monte Carlo based on the actual MAP estimate given in the MAPestP and MAPestW arguments (unlike BICM1, no approximation of the MAP estimate from the MCMC sample).

# Author(s)

Cristina Mollica and Luca Tardella

## References

Mollica, C. and Tardella, L. (2017). Bayesian Plackett-Luce mixture models for partially ranked data. *Psychometrika*, **82**(2), pages 442–458, ISSN: 0033-3123, <doi:10.1007/s11336-016-9530-0>.

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Gelman, A., Carlin, J. B., Stern, H. S. and Rubin, D. B. (2004). Bayesian data analysis. Chapman & Hall/CRC, Second Edition, ISBN: 1-58488-388-X. New York.

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summary.gsPLMIX

Summary of the Gibbs sampling procedure for a Bayesian mixture of Plackett-Luce models

## Description

summary method for class gsPLMIX. It provides summary statistics and credible intervals for the Gibbs sampling simulation of a Bayesian mixture of Plackett-Luce models.

#### Usage

```
## S3 method for class 'gsPLMIX'
summary(
   object,
   quantiles = c(0.025, 0.25, 0.5, 0.75, 0.975),
   hpd_prob = 0.95,
   digits = 2,
   ...
)
```

## Arguments

object	Object of class gsPLMIX returned by the gibbsPLMIX function.
quantiles	Numeric vector of quantile probabilities.
hpd_prob	Numeric scalar in the grid of values spanning the interval $(0,1)$ by 0.05, giving the posterior probability content of the HPD intervals. Supplied values outside the grid are rounded.
digits	Number of decimal places for rounding the posterior summaries.
	Further arguments passed to or from other methods (not used).

## Details

Posterior summaries include means, standard deviations, naive standard errors of the means (ignoring autocorrelation of the chain) and time-series standard errors based on an estimate of the spectral density at 0. They correspond to the statistics element of the output returned by the summary.mcmc function of the coda package. Highest posterior density (HPD) intervals are obtained by recalling the HPDinterval function of the coda package.

## Value

A list of summary statistics for the gsPLMIX class object:

statistics	Numeric matrix with posterior summaries in each row (see 'Details').	
quantiles	Numeric matrix with posterior quantiles at the given quantiles probabilities in each row.	
HPDintervals	Numeric matrix with 100*hpd_prob% HPD intervals in each row.	
Modal_orderings		
	Numeric $G \times K$ matrix with the estimated posterior modal orderings of each mixture component.	
call	The matched call.	

# Author(s)

Cristina Mollica and Luca Tardella

# References

Plummer, M., Best, N., Cowles, K. and Vines, K. (2006). CODA: Convergence Diagnosis and Output Analysis for MCMC, *R News*, **6**, pages 7–11, ISSN: 1609-3631.

# See Also

summary.mcmc and HPDinterval

# Examples

```
data(d_carconf)
GIBBS <- gibbsPLMIX(pi_inv=d_carconf, K=ncol(d_carconf), G=3, n_iter=30, n_burn=10)
## Summary of the Gibbs sampling procedure
summary(GIBBS)</pre>
```

summary.mpPLMIX	Summary of the MAP estimation for a Bayesian mixture of Plackett-
	Luce models

# Description

summary method for class mpPLMIX. It provides summaries for the MAP estimation of a Bayesian mixture of Plackett-Luce models.

# Usage

## S3 method for class 'mpPLMIX'
summary(object, digits = 2, ...)

## Arguments

object	Object of class <code>mpPLMIX</code> returned by the <code>mapPLMIX</code> or <code>mapPLMIX_multistart</code> function.
digits	Number of decimal places for rounding the summaries.
	Further arguments passed to or from other methods (not used).

# Value

A list of summaries for the mpPLMIX class object:

MAP_w	Numeric vector with the MAP estimates of the $G$ mixture weights. Returned only when when $G > 1$ .	
MAP_p	Numeric $G \times K$ matrix with the MAP estimates of the component-specific support parameters.	
MAP_modal_orderings		
	Numeric $G \times K$ matrix with the estimated modal orderings of each mixture component.	
group_distr	Numeric vector with the relative frequency distribution of the mixture component memberships based on MAP allocation. Returned only when when $G > 1$ .	
perc_conv_rate	Numeric scalar with the percentage of MAP algorithm convergence over the multiple starting points. Returned only when summary.mpPLMIX is applied to the output of the mapPLMIX_multistart function.	

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## unit\_to\_freq

## Author(s)

Cristina Mollica and Luca Tardella

#### Examples

```
## Summary of the MAP procedure with a single starting point
data(d_carconf)
MAP <- mapPLMIX(pi_inv=d_carconf, K=ncol(d_carconf), G=3)
summary(MAP)
## Summary of the MAP procedure with 5 starting points
MAP_multi <- mapPLMIX_multistart(pi_inv=d_carconf, K=ncol(d_carconf), G=3, n_start=5)
summary(MAP_multi)
```

unit\_to\_freq Frequency distribution from the individual rankings/orderings

#### Description

Construct the frequency distribution of the distinct observed sequences from the dataset of individual rankings/orderings.

#### Usage

```
unit_to_freq(data)
```

## Arguments

data Numeric  $N \times K$  data matrix of observed individual sequences.

# Value

Numeric matrix of the distinct observed sequences with the corresponding frequencies indicated in the last (K + 1)-th column.

# Author(s)

Cristina Mollica and Luca Tardella

```
## Frequency distribution for the APA top-ordering dataset
data(d_apa)
unit_to_freq(data=d_apa)
```

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